Interference Localisation Methods using Direct Position Determination Concept

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Introduction

- GNSS signals are inherently weak
- Spurious transmissions and intentional jammers in the GNSS band threatens safety critical applications that depends on GNSS

- A network of phased array sensors tuned to the GNSS band can be used to detect jammers.
Jammer Characteristics

- **Narrowband**
  - Strong jammer signal strength will affect receiver performance
  - Can be detected using AOA

- **Wideband**
  - Weak jammer signal strength is sufficient to affect receiver performance
  - Can be detected using TDOA and AOA
Introduction

- AOA: Angle of Arrival utilising phased array processing
- TDOA: Time Difference of Arrival utilising cross correlation
- Geo-localisation of jammer
  - AOA: Intersection of lines
  - TDOA: Intersection of hyperbolas
Direct Position Determination (DPD)

- A signal processing technique to directly localise the jammer in the position domain
- Aims to combine signal energy from all antenna elements in the network
- Provides better position resolution than conventional methods
Existing DPD Approaches

• Most DPD approaches models narrowband signals (e.g. DPD, LOST, LOST-FIND, HR-DPD)
• Assumes wideband signal as a combination of multiple narrowband channels
• These DPD algorithms does not exploit good cross-correlation properties of wideband signals

\[
\begin{align*}
\mathbf{r}_e(j, k) &= \sum_{q=1}^{Q} b_{eq} a_{eq}(p_q) s_q(j, k) e^{-i\omega j [\tau_e(p_q) + t_q^{(0)}]} + \mathbf{n}_e(j, k), \\
\tilde{s}_q(j, k) &\triangleq s_q(j, k) e^{-i\omega_j t_q^{(0)}}, \\
\tilde{a}_e(j, p_q, b_{eq}) &\triangleq b_{eq} a_{eq}(p_q) e^{-i\omega_j \tau_e(p_q)}.
\end{align*}
\]
Taxonomy of DPD Methods

Direct Position Determination

Narrowband
- MUSIC / Subspace
- MVDR

Wideband
- MUSIC / Subspace
- MVDR
TARGET 1/2

- Signal model:

\[
\begin{align*}
    r_i(t) &= \sum_{q=1}^{Q} \alpha_{i,q} a_i(p_q) s_q(t - \tau_{i,q}(p_q)) + n_i(t) \\
    &= A_i \Omega_is_i(t) + n_i(t)
\end{align*}
\]

- Eigen-decomposition

Correct eigendecomposition requires \( Q < M \)

- Form noise subspace

\[
\begin{align*}
    x_{i,j}(t, \tau) &= [r_i^T(t) \ r_j^T(t + \tau)]^T \\
    R_x(\tau) &= \mathbb{E}_t \left( x_{i,j}(t, \tau)x_{i,j}^H(t, \tau) \right) \\
    R_x(\tau) &= \begin{bmatrix} R_{ii} & R_{ij}(\tau) \\ R_{ji}(-\tau) & R_{jj} \end{bmatrix} + R_n \\
    R_{ii}^+ &= \Pi_i \Sigma_i^{-1} \Pi_i^n \\
    R_{ii} &= \Pi_i \Sigma_i^n \\
    G_{ij}(\tau) &= I_M - R_{ii}^+ R_{ij}(-\tau) R_{jj}^+ R_{ji}(\tau)
\end{align*}
\]
TARGET 2/2

• Cost function:

\[ \hat{p}_q = \arg\min_{p \in P} \frac{1}{L(L-1)} \sum_{i \neq j} J_{T,ij}(p_q) \quad \forall i, j \in [1, L] \]

where

\[ J_{T,ij}(p_q) = \frac{\alpha^H_i(p_q) G_{ij}(\tau(p_q)) \alpha_i(p_q)}{\alpha^H_i(p_q) \alpha_i(p_q)} \]

• Gridded position domain search:

(left) X-Y, (middle) Y-Z and (right) X-Z domain plot of the test statistic (z-axis) vs position space (x,y-axis)
Limitations of TARGET

• Requires assumed knowledge of Q
• Limited number of detectable sources
• Lack sensitivity
  – Does not fully utilise signal energy from all antenna elements within the array
Cross-correlation DPD (1/2)

- Global Covariance Matrix

- Modified Global Covariance Matrix
Cross-correlation DPD (2/2)

• Eigen decomposition and cost function

\[ J_{ccDPD}(\rho) = \lambda_{\text{min}} \left( (A^H(\rho)A(\rho))^{-1} A^H(\rho)\tilde{F}_y(\rho)A(\rho) \right) \]

where

\[ \tilde{F}_y(\rho) = I_{LM} - H(\rho)G(\rho)G^H(\rho)H^H(\rho) \]

\[ G(\rho) = e_{\tilde{R}_y}(\rho)\Sigma_{\tilde{R}_y} e_{\tilde{R}_y}^H(\rho) \]

\[ H(\rho) = e_{R_y}(\rho)\Sigma_{R_y}^{-1} e_{R_y}^H(\rho) \]

and

\[ A(\rho) = \begin{bmatrix} a_1(\rho) & \cdots & 0_{M \times 1} \\ \vdots & \ddots & \vdots \\ 0_{M \times 1} & \cdots & a_L(\rho) \end{bmatrix}_{LM \times L} \]
Multiple Jammer

<table>
<thead>
<tr>
<th>TARGET</th>
<th>ccDPD</th>
</tr>
</thead>
</table>

SNR = 0dB
Heavy Background 8x GNSS Signals

<table>
<thead>
<tr>
<th>TARGET</th>
<th>ccDPD</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="TARGET Diagram" /></td>
<td><img src="image2.png" alt="ccDPD Diagram" /></td>
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</table>

SNR = -10dB
**Large Number of Sources (Ns = 12)**

<table>
<thead>
<tr>
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<th>ccDPD</th>
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<tbody>
<tr>
<td><img src="image1.png" alt="TARGET Image" /></td>
<td><img src="image2.png" alt="ccDPD Image" /></td>
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**SNR = -10dB**
### Performance Evaluation

<table>
<thead>
<tr>
<th>Multiple Jammer</th>
<th>Heavy Background GNSS Signals</th>
<th>Large number of sources</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th><strong>SNR (dB)</strong></th>
<th><strong>RMSE (m)</strong></th>
<th><strong>SNR (dB)</strong></th>
<th><strong>RMSE (m)</strong></th>
<th><strong>SNR (dB)</strong></th>
<th><strong>RMSE (m)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td>2</td>
<td>-10</td>
<td>30</td>
<td>-10</td>
<td>30</td>
</tr>
<tr>
<td>-5</td>
<td>1.5</td>
<td>-5</td>
<td>25</td>
<td>-5</td>
<td>25</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>0.5</td>
<td>5</td>
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<td>5</td>
<td>5</td>
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<tr>
<td>10</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>10</td>
<td>0</td>
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</tbody>
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- **SNR (dB)**: Signal-to-Noise Ratio (in decibels)
- **RMSE (m)**: Root Mean Square Error (in meters)
# Field Data Results

<table>
<thead>
<tr>
<th></th>
<th>TARGET</th>
<th>ccDPD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>East (m)</strong></td>
<td>-420</td>
<td>-420</td>
</tr>
<tr>
<td></td>
<td>-400</td>
<td>-400</td>
</tr>
<tr>
<td></td>
<td>-380</td>
<td>-380</td>
</tr>
<tr>
<td></td>
<td>-360</td>
<td>-360</td>
</tr>
<tr>
<td></td>
<td>-340</td>
<td>-340</td>
</tr>
<tr>
<td><strong>North (m)</strong></td>
<td>340</td>
<td>340</td>
</tr>
<tr>
<td></td>
<td>360</td>
<td>360</td>
</tr>
<tr>
<td></td>
<td>380</td>
<td>380</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td></td>
<td>420</td>
<td>420</td>
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**SNR = -10dB**
Conclusion

• Derived a taxonomy and compared various DPD approaches
• Proposed ccDPD method has superior SNR sensitivity in comparison to recent methods
• Proposed ccDPD method can localise more sources than TARGET
Questions?

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