

# Analysis of Double-Differenced Multi-GNSS Inter-System Biases for Overlapping and Mixed Frequencies

*Shuyang Cheng, Jinling Wang and Wenhao Zhang*

School of Civil and Environmental Engineering, University of New South Wales,  
Sydney, NSW 2052, Australia

Phone: +61 416 866 545 Fax: +61 2 9385 6139 Email: [shuyang.cheng@student.unsw.edu.au](mailto:shuyang.cheng@student.unsw.edu.au)

## ABSTRACT

Compatibility and interoperability among multi-GNSS constellations are necessary for reliable multi-GNSS positioning and navigation, thus, attracting great attention from the satellite navigation community over recent years. It has been well known that Double-Differenced (DD) integer ambiguities between satellites in the same constellation are resolvable. If the integer carrier phase ambiguities between satellites in different constellations are to be resolved, multi-GNSS Inter-System Bias (ISB) should be taken into consideration. Most of current studies focus on ISB estimation and statistical analysis for overlapping frequencies of different constellations, such as GPS-Galileo (L1-E1, L5-E5a) or Galileo-BDS (E5b-B2). ISB estimation for mixed frequencies is still challenging and rarely investigated.

In this paper, we will design procedures to estimate multi-GNSS code and carrier phase ISBs towards the goal to resolve the DD integer ambiguities between any satellites within the multi-constellation GNSS. In addition, the statistical characteristics of these biases for both overlapping and mixed frequencies will be analysed. Data from several ultra-short and zero baselines are used to estimate the DD ISBs for three types of commercial GNSS receivers. Experimental results indicate that for the baselines with the same receiver type, code and phase ISBs have long-term stability for both overlapping and mixed frequencies. Although the code and phase ISBs may have small variations for the baselines with different receiver types, these variations are very slow and smooth, so they can be properly modelled.

**KEYWORDS:** Inter-System Biases, Multi-GNSS, Inter-System Double-Differencing, Statistical Analysis

## 1. INTRODUCTION

Over the past three decades, Global Navigation Satellite System (GNSS), which includes GPS from U.S., GLONASS from Russia, BDS from China and Galileo system from the European Union, has been a reliable, versatile and widely-used positioning technology in precise navigation applications. With the modernization of GPS and GLONASS as well as the development of Galileo and BDS, more satellites, frequencies and trackable signals will be accessible to a broad range of users, which brings both opportunities and challenges for high-precision positioning. Since multi-frequency multi-GNSS is the future trend, the potential and benefit of multi-constellation observations need to be fully exploited to improve the performance of GNSS navigation systems. It has been proven that the fusion of measurements from different GNSS constellations can enhance the model geometry strength and provide a significant improvement to the positioning performance of Precise Point Positioning (PPP) (Li et al. 2015) as well as Real Time Kinematic (RTK) (Odolinski et al. 2015), in terms of accuracy, reliability, availability and initialization time.

Compatibility and interoperability among multi-GNSS constellations are necessary for reliable multi-GNSS positioning and becoming the major concern in the satellite navigation community. According to Gibbons (2006), a main aspect of GNSS interoperability is to achieve better performance at the user level via the combination of observations from two or more GNSS constellations. It has been well known that Double-Differenced (DD) integer ambiguities between satellites in the same constellation are resolvable. In this case, one reference satellite is selected for each GNSS system to form DD ambiguities in ambiguity resolution, which is called inner-system differencing or loosely-coupled model. However, the conventional loosely-coupled DD model may not work effectively in severe observation environment, such as urban areas or canyons where signals are easily blocked so that few satellites are visible in each GNSS system. If the integer carrier phase ambiguities between satellites in different constellations can be formed and resolved, the geometry strength of DD positioning model can be improved and the availability and reliability of relative positioning will increase. In that case, a common pivot satellite is chosen for all the GNSS systems to form both inner-system and inter-system DD ambiguities, which is referred to as inter-system differencing or tightly-coupled model. When the tightly-coupled DD model is introduced, the interoperability of GNSS systems is expected to be improved. Moreover, multi-GNSS Inter-System Bias (ISB) needs to be taken into consideration (Odijk and Teunissen 2013; Paziewski and Wielgosz 2015).

Odijk and Teunissen (2013) analysed the magnitude and stability of code and phase DD ISBs between GPS and Galileo In-Orbit Validation Element (GIOVE) at the overlapping L1-E1 and L5-E5a frequencies. The results demonstrated that the ISBs are close to zero and can be neglected in tightly-coupled DD model for baselines consisting of identical receiver types. But for baselines formed by different receiver types, the ISBs are usually significant but very stable in a long term thus they can be calibrated by a priori correction. In addition, improved ambiguity resolution performance can be obtained after ISBs are properly calibrated in tightly-coupled DD model. Similar conclusions can also be found in the work by Paziewski and Wielgosz (2015), but they treated the ISB parameters as constant in longer sessions rather than epoch-varying and achieved better repeatability than single-epoch solutions. Wu et al. (2017) evaluated the interoperability of BDS with Galileo and investigated the code and phase DD ISBs between the BDS B2 and Galileo E5b signals. Liu et al. (2017) examined the code DD ISBs of GLONASS/BDS/Galileo with regard to GPS and proved that the positioning results are obviously improved after calibrating the ISBs, compared with traditional

Differential GNSS (DGNSS) model, especially in harsh environments.

Currently, most research work only focuses on ISB estimation and statistical analysis for overlapping frequencies of different GNSS systems, such as GPS/Quasi-Zenith Satellite System (QZSS) L1-Galileo E1, GPS/QZSS L5-Galileo E5a, and BDS B2-Galileo E5b (Odolinski et al. 2015; Wu et al. 2017). In order to take full advantage of the measurements from all the satellites and frequencies, tightly-coupled DD model for mixed frequencies needs to be studied, but ISB estimation for mixed frequencies is still challenging and rarely investigated.

In this paper, we will design the procedure to estimate multi-GNSS code and carrier phase ISBs towards the goal to resolve the DD integer ambiguities between any satellites within the multi-constellation GNSS. The structure of the paper is as follows: In Section 2, the basic observation equations of multi-GNSS positioning model and the frequencies of different GNSS signals are presented. In Section 3, tightly-coupled DD model is introduced with the estimation approach of ISBs for both overlapping and mixed frequencies. In Section 4, specific numerical analysis is conducted to analyse the statistical characteristics of the estimated ISBs. Finally, some conclusions are derived and the future research interest is proposed.

## 2. MULTI-GNSS OBSERVATION MODEL AND FREQUENCY

For a specific pair of satellite  $s$  and receiver  $r$ , the observation equations of raw GNSS pseudorange  $P$  and carrier phase  $L$  are written as:

$$P_{r,i}^{s,G} = \rho_r^{s,G} + cdt_r^G - cdt^{s,G} + T_r^{s,G} + I_{r,i}^{s,G} + D_{r,i}^G - D_i^{s,G} + \varepsilon_{pi}^{s,G} \quad (1)$$

$$L_{r,i}^{s,G} = \rho_r^{s,G} + cdt_r^G - cdt^{s,G} + T_r^{s,G} - I_{r,i}^{s,G} + \lambda_i^G (N_i^G + \varphi_{r,i}^G - \varphi_i^{s,G}) + \varepsilon_{Li}^{s,G} \quad (2)$$

$$P_{r,i}^{s,*} = \rho_r^{s,*} + (cdt_r^G + ISTB^*) - cdt^{s,*} + T_r^{s,*} + I_{r,i}^{s,*} + D_{r,i}^* - D_i^{s,*} + \varepsilon_{pi}^{s,*} \quad (3)$$

$$L_{r,i}^{s,*} = \rho_r^{s,*} + (cdt_r^G + ISTB^*) - cdt^{s,*} + T_r^{s,*} - I_{r,i}^{s,*} + \lambda_i^* (N_i^* + \varphi_{r,i}^* - \varphi_i^{s,*}) + \varepsilon_{Li}^{s,*} \quad (4)$$

where subscript  $i$  represents the frequency band of GNSS observation; superscript  $G$  stands for GPS satellites and superscript  $*$  is the symbol for BDS or Galileo satellites;  $\rho_r^s$  denotes the geometric distance between satellite  $s$  and receiver  $r$ ;  $c$  is the speed of light in vacuum;  $dt_r^G$  is the receiver clock offset referenced to GPS Time;  $dt^s$  is the satellite clock offset;  $ISTB$  is the inter-system time bias with regard to GPS;  $T_r^s$  is the slant tropospheric delay and  $I_{r,i}^s$  is the slant ionospheric delay on frequency band  $i$ ;  $\lambda_i$  is the wavelength of frequency band  $i$ ;  $N_i$  is the integer ambiguity on frequency band  $i$ ;  $D_{r,i}$  and  $D_i^s$  are the receiver and satellite code instrumental delays on frequency band  $i$ ;  $\varphi_{r,i}$  and  $\varphi_i^s$  are the receiver and satellite phase instrumental delays combined with initial phase biases on frequency band  $i$ ;  $\varepsilon_p$  and  $\varepsilon_L$  are measurement noises and multipath effects of pseudorange and carrier phase.

GPS L1/L2/L5, BDS B1/B2 and Galileo E1/E5a signals are used in this study. The frequencies of these signals are summarized in Table 1. Signals with the same frequency are marked in bold. GPS L1-Galileo E1 and GPS L5-Galileo E5a ISBs are estimated in the overlapping frequency scenario while GPS L1-BDS B1 and GPS L2-BDS B2 ISBs are derived in the mixed frequency scenario.

GNSS constellation	Band	Frequency (MHz)
GPS	<b>L1</b>	<b>1575.420</b>
	L2	1227.600
	<b>L5</b>	<b>1176.450</b>
Galileo	<b>E1</b>	<b>1575.420</b>
	<b>E5a</b>	<b>1176.450</b>
BDS	B1	1561.098
	B2	1207.140

**Table 1.** Multi-GNSS signals used in this study

### 3. DD ISB ESTIMATION

In this section, we will present the observation equations of GPS-Galileo (overlapping frequency) and GPS-BDS (mixed frequency) ISB estimation. As zero or ultra-short baselines are used to estimate DD ISBs in our study, the satellite-specific and receiver-specific systematic errors can be eliminated. In addition, both the ionospheric delays and tropospheric delays can also be ignored. In tightly-coupled DD model, a common GPS pivot satellite is chosen for all the GNSS systems to form both inner-system (GPS-GPS) and inter-system (GPS-Galileo or GPS-BDS) DD ambiguities.

#### 3.1 Overlapping Frequency Scenario

The observation equations of Between-Satellite Single-Differenced (BSSD) GPS/Galileo model are expressed as:

$$P_{r,i}^{G0G} = \rho_r^{G0G} - cdt^{G0G} + T_r^{G0G} + I_{r,i}^{G0G} - D_i^{G0G} + \varepsilon_{pi}^{G0G} \quad (5)$$

$$L_{r,i}^{G0G} = \rho_r^{G0G} - cdt^{G0G} + T_r^{G0G} - I_{r,i}^{G0G} + \lambda_i(N_i^{G0G} - \varphi_i^{G0G}) + \varepsilon_{Li}^{G0G} \quad (6)$$

$$P_{r,i}^{G0E} = \rho_r^{G0E} + ISTB^E - cdt^{G0E} + T_r^{G0E} + I_{r,i}^{G0E} + D_{r,i}^{G0E} - D_i^{G0E} + \varepsilon_{pi}^{G0E} \quad (7)$$

$$L_{r,i}^{G0E} = \rho_r^{G0E} + ISTB^E - cdt^{G0E} + T_r^{G0E} - I_{r,i}^{G0E} + \lambda_i(N_i^{G0E} + \varphi_{r,i}^{G0E} - \varphi_i^{G0E}) + \varepsilon_{Li}^{G0E} \quad (8)$$

where  $G0$  is the common GPS reference satellite and  $E$  is the Galileo satellite;  $\lambda_i$  is the wavelength of overlapping frequency;  $\varphi^{G0G} = \varphi^{s,G} - \varphi^{s,G0}$ ;  $\varphi^{G0E} = \varphi^{s,E} - \varphi^{s,G0}$ .

For base receiver  $b$  and rover receiver  $r$ , ignoring DD atmospheric delays, DD GPS/Galileo observation model is written as:

$$P_{br,i}^{G0G} = \rho_{br}^{G0G} + \varepsilon_{br,pi}^{G0G} \quad (9)$$

$$L_{br,i}^{G0G} = \rho_{br}^{G0G} + \lambda_i N_{br,i}^{G0G} + \varepsilon_{br,Li}^{G0G} \quad (10)$$

$$P_{br,i}^{G0E} = \rho_{br}^{G0E} + D_{br,i}^{G0E} + \varepsilon_{br,pi}^{G0E} \quad (11)$$

$$L_{br,i}^{GOE} = \rho_{br}^{GOE} + \lambda_i(N_{br,i}^{GOE} + \varphi_{br,i}^{GOE}) + \varepsilon_{br,Li}^{GOE} \quad (12)$$

where  $D_{br,i}^{GOE}$  and  $\varphi_{br,i}^{GOE}$  are the GPS-Galileo DD code and phase ISBs, respectively;  $\bigcirc_{br}^{GOG} = \bigcirc_r^{GOG} - \bigcirc_b^{GOG}$ ,  $\bigcirc_{br}^{GOE} = \bigcirc_r^{GOE} - \bigcirc_b^{GOE}$ .

It can be found that  $\varphi_{br,i}^{GOE}$  is related to  $N_{br,i}^{GOE}$ , so it is not possible to simultaneously estimate the DD phase ISB parameter and the DD inter-system ambiguities with respect to GPS reference satellite due to the rank deficiency. According to Odijk and Teunissen (2013), the DD inter-system ambiguity with respect to GPS reference satellite can be reparameterized into the ambiguity relative to the virtual Galileo reference satellite, plus the ambiguity of the virtual Galileo reference satellite with respect to the GPS reference satellite, which yields:

$$N_{br,i}^{GOE} = N_{br,i}^E - N_{br,i}^{G0} = (N_{br,i}^E - N_{br,i}^{E0}) + (N_{br,i}^{E0} - N_{br,i}^{G0}) = N_{br,i}^{E0E} + N_{br,i}^{G0E0} \quad (13)$$

$$L_{br,i}^{GOE} = \rho_{br}^{GOE} + \lambda_i(N_{br,i}^{E0E} + N_{br,i}^{G0E0} + \varphi_{br,i}^{GOE}) + \varepsilon_{br,Li}^{GOE} = \rho_{br}^{GOE} + \lambda_i(N_{br,i}^{E0E} + \varnothing_{br,i}^{GOE}) + \varepsilon_{br,Li}^{GOE} \quad (14)$$

where  $E0$  is the virtual Galileo reference satellite;  $\varnothing_{br,i}^{GOE}$  is the estimable GPS-Galileo DD phase ISB, and  $\varnothing_{br,i}^{GOE} = \varphi_{br,i}^{GOE} + N_{br,i}^{G0E0}$ .

### 3.2 Mixed Frequency Scenario

The inner-system (GPS-GPS) BSSD observation equations are same to Equations (5) and (6). The inter-system (GPS-BDS) BSSD observation equations are expressed as:

$$P_{r,i}^{G0C} = \rho_r^{G0C} + ISTB^C - cdt^{G0C} + T_r^{G0C} + I_{r,i}^{G0C} + D_{r,i}^{G0C} - D_i^{G0C} + \varepsilon_{pi}^{G0C} \quad (15)$$

$$L_{r,i}^{G0C} = \rho_r^{G0C} + ISTB^C - cdt^{G0C} + T_r^{G0C} - I_{r,i}^{G0C} + \lambda_i^C(N_i^C + \varphi_{r,i}^C - \varphi_i^{s,C}) - \lambda_i^G(N_i^{G0} + \varphi_{r,i}^{G0} - \varphi_i^{s,G0}) + \varepsilon_{Li}^{G0C} \quad (16)$$

where  $C$  is the BDS satellite;  $\lambda_i^G$  and  $\lambda_i^C$  are the wavelength of corresponding frequency of GPS and BDS satellites, respectively;  $\bigcirc^{G0C} = \bigcirc^{s,C} - \bigcirc^{s,G0}$ .

The inner-system (GPS-GPS) DD observation equations are same to Equations (9) and (10). For base receiver  $b$  and rover receiver  $r$ , ignoring DD atmospheric delays and correcting the BDS inter-satellite-type bias (Nadarajah et al. 2015) in advance, the inter-system (GPS-BDS) DD observation equations are written as:

$$P_{br,i}^{G0C} = \rho_{br}^{G0C} + D_{br,i}^{G0C} + \varepsilon_{br,pi}^{G0C} \quad (17)$$

$$L_{br,i}^{G0C} = \rho_{br}^{G0C} + \lambda_i^C \varphi_{br,i}^{G0C} + \lambda_i^G N_{br,i}^{G0C} + (\lambda_i^C - \lambda_i^G) N_{br,i}^{G0} + \varepsilon_{br,Li}^{G0C} \quad (18)$$

where  $D_{br,i}^{G0C}$  and  $\varphi_{br,i}^{G0C}$  are the GPS-BDS DD code and phase ISBs, respectively;  $\varphi_{br,i}^{G0C} = \varphi_{br,i}^C - (\lambda_i^G/\lambda_i^C) \varphi_{br,i}^{G0}$ ,  $\bigcirc_{br}^{G0C} = \bigcirc_r^{G0C} - \bigcirc_b^{G0C}$ .

In the same way as Equation (13), we can reparameterize the DD inter-system ambiguity as:

$$N_{br,i}^{G0C} = N_{br,i}^C - N_{br,i}^{G0} = (N_{br,i}^C - N_{br,i}^{C0}) + (N_{br,i}^{C0} - N_{br,i}^{G0}) = N_{br,i}^{C0C} + N_{br,i}^{G0C0} \quad (19)$$

where  $C0$  is the virtual BDS reference satellite.

Thus Equation (18) can be rewritten as:

$$\begin{aligned} L_{br,i}^{G0C} &= \rho_{br}^{G0C} + \lambda_i^C \varphi_{br,i}^{G0C} + \lambda_i^C N_{br,i}^{G0C0} + (\lambda_i^C - \lambda_i^G) N_{br,i}^{G0} + \lambda_i^C N_{br,i}^{C0C} + \varepsilon_{br,Li}^{G0C} \\ &= \rho_{br}^{G0C} + \lambda_i^C (N_{br,i}^{C0C} + \varnothing_{br,i}^{G0C}) + \varepsilon_{br,Li}^{G0C} \end{aligned} \quad (20)$$

where  $\varnothing_{br,i}^{G0C}$  is the estimable GPS-BDS DD phase ISB, and  $\varnothing_{br,i}^{G0C} = \varphi_{br,i}^{G0C} + N_{br,i}^{G0C0} + (1 - \lambda_i^G / \lambda_i^C) N_{br,i}^{G0}$ .

### 3.3 Continuity of Estimable DD ISB

It is obvious that the estimable DD phase ISB will absorb the DD integer ambiguity in both overlapping frequency and mixed frequency scenarios. Now that only the fractional part of phase ISB will affect the DD ambiguity resolution, we will just analyse the stability of the fractional phase ISBs in the next section. From the form of estimable DD phase ISB, we can see that the fractional GPS-Galileo ISB will not change if the GPS reference satellite varies. But things are different for mixed frequency scenario, thus, the continuity of GPS-BDS phase ISB needs to be accounted for in the real-time epoch-to-epoch estimation process.

Assuming that the GPS pivot satellite changes from  $G0$  to  $G1$  between two consecutive epochs  $T_0$  and  $T_1$ , the GPS-BDS phase ISB can be transformed as follows (Gao et al. 2017).

At epoch  $T_0$ , the estimable phase ISB is:

$$\varnothing_{br,i}^{G0C}(T_0) = \varphi_{br,i}^{G0C} + N_{br,i}^{G0C0} + (1 - \lambda_i^G / \lambda_i^C) N_{br,i}^{G0} \quad (21)$$

At epoch  $T_1$ , the estimable phase ISB is:

$$\varnothing_{br,i}^{G0C}(T_1) = \varphi_{br,i}^{G0C} + N_{br,i}^{G1C0} + (1 - \lambda_i^G / \lambda_i^C) N_{br,i}^{G1} \quad (22)$$

The estimable phase ISB at epoch  $T_1$  can be transformed to:

$$\begin{aligned} \tilde{\varnothing}_{br,i}^{G0C}(T_1) &= \varnothing_{br,i}^{G0C}(T_1) - (1 - \lambda_i^G / \lambda_i^C) N_{br,i}^{G0G1} \\ &= \varphi_{br,i}^{G0C} + N_{br,i}^{G1C0} + (1 - \lambda_i^G / \lambda_i^C) \left[ (N_{br,i}^{G1} - (N_{br,i}^{G1} - N_{br,i}^{G0})) \right] \\ &= \varphi_{br,i}^{G0C} + N_{br,i}^{G1C0} + (1 - \lambda_i^G / \lambda_i^C) N_{br,i}^{G0} \end{aligned} \quad (23)$$

By Equation (23), we can obtain the continuous fractional phase ISB between two epochs.

## 4. NUMERICAL ANALYSIS

### 4.1 Experimental Data and Processing Strategy

One-week GNSS data of zero and ultra-short baselines collected on the campus of Curtin University, Perth, Australia, were used in this study. The data were collected from DOY 339, 2016 to DOY 345, 2016 with the sampling rate of 30 s. The information of these baselines is shown in Table 2. All the stations use the same type of antenna: ‘TRM 59800.00 SCIS’.

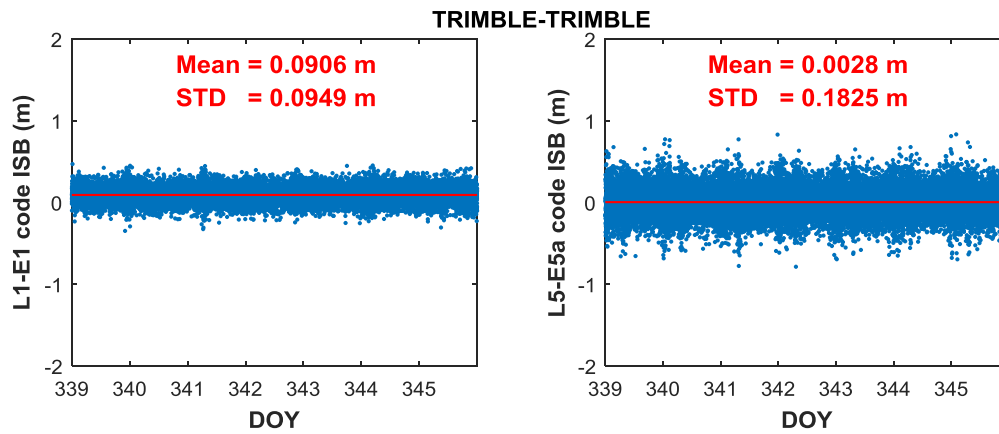
Among these three baselines, the first one consists of identical receiver type, while the other two are formed by different receiver types.

Baseline	Base Receiver Type	Rover Receiver Type	Baseline Length / m
CUT0-CUT2	TRIMBLE NETR9	TRIMBLE NETR9	0
CUT0-CUT1	TRIMBLE NETR9	SEPTENTRIO POLARX4	0
CUT0-CUAA	TRIMBLE NETR9	JAVAD TRE_G3TH_8	8.42

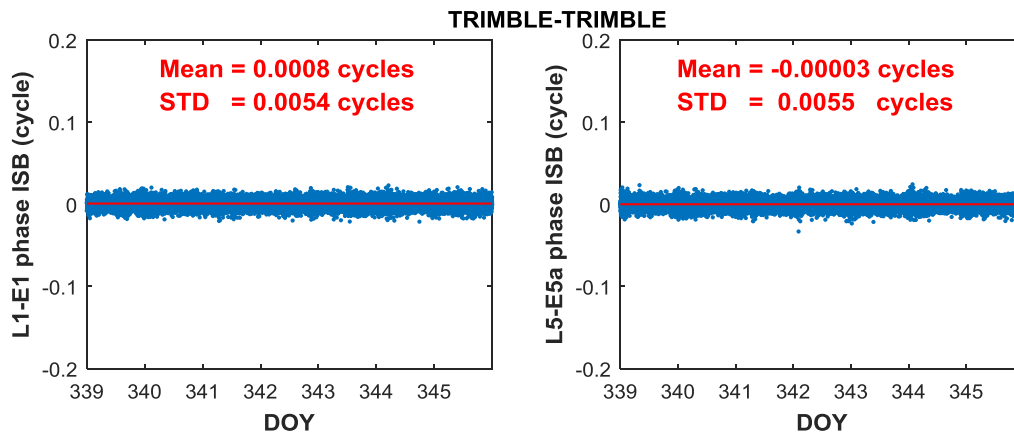
**Table 2.** Information of baselines used in the experiments

In order to evaluate the variation of DD ISBs with time, both code and phase ISBs are estimated epoch-by-epoch. In addition, the baseline components were fixed to the truth value to enhance the solution. For convenience, only the fractional part of estimable phase ISB is analysed. Equation (23) is used to adjust the estimable phase ISB and derive the continuous fractional phase ISB between GPS and BDS.

#### 4.2 GPS-Galileo DD ISB Stability Analysis



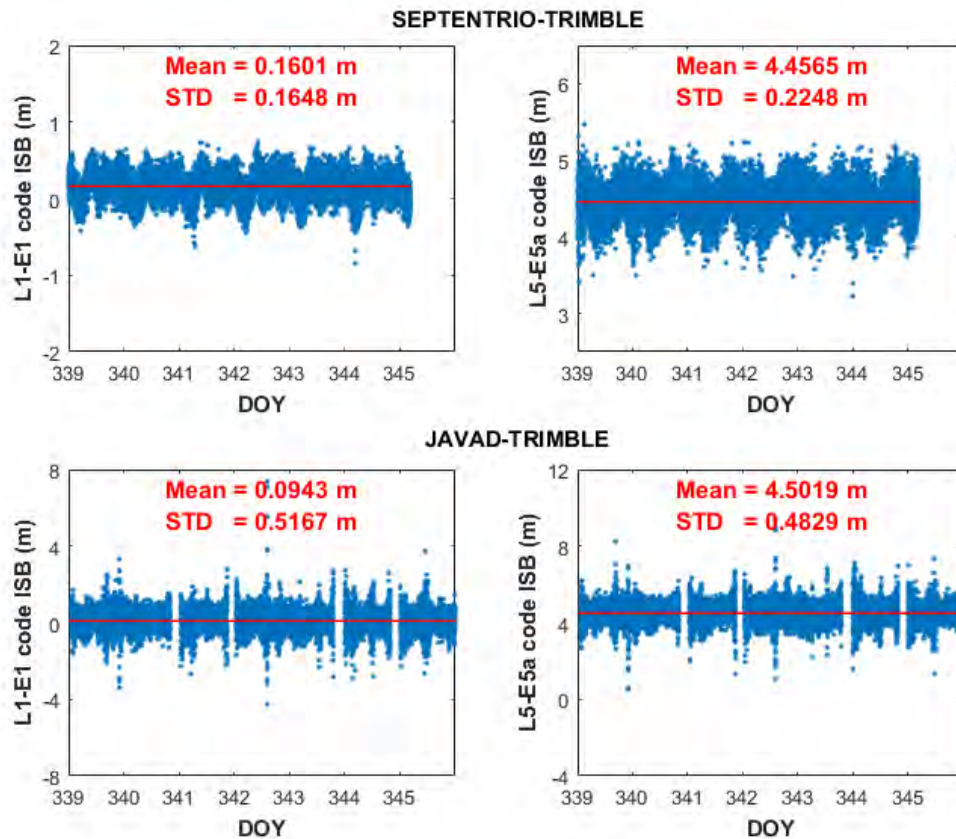
**Figure 1.** Time series of GPS-Galileo (L1-E1 and L5-E5a) code ISBs for the baseline with the same receiver type: ‘TRIMBLE’



**Figure 2.** Time series of GPS-Galileo (L1-E1 and L5-E5a) phase ISBs for the baseline with the same receiver type: ‘TRIMBLE’

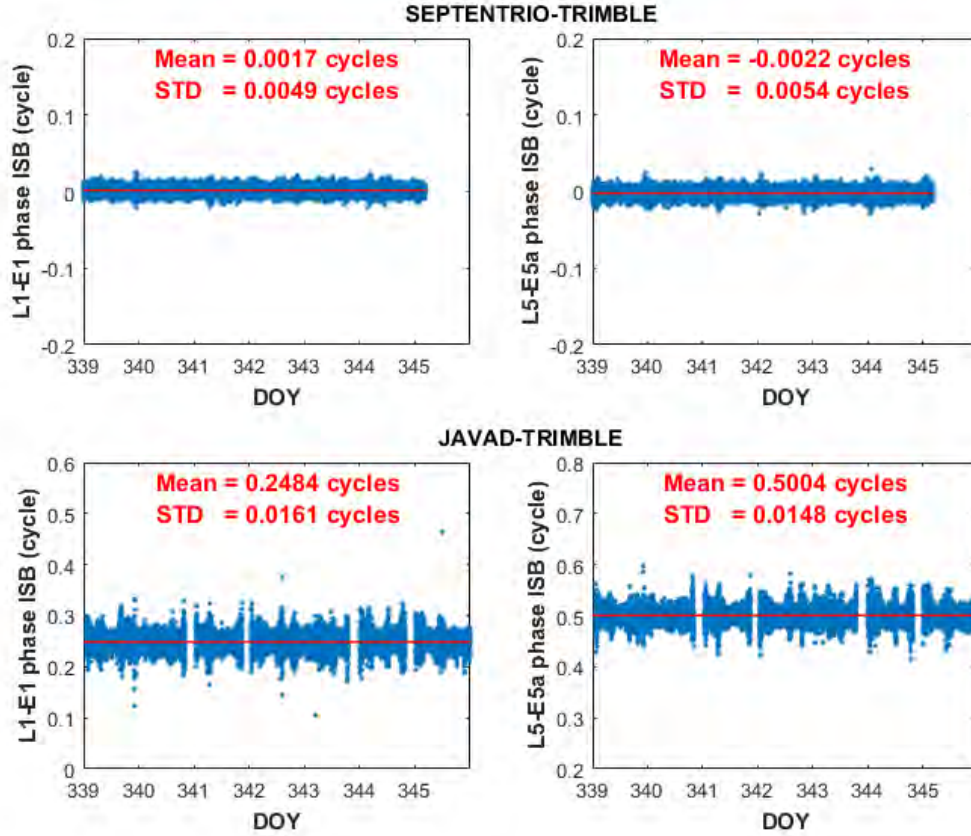
Figures 1 and 2 depict the time series of GPS-Galileo (L1-E1 and L5-E5a) code and phase ISBs, respectively, for the baseline with the same receiver type: ‘TRIMBLE’. The blue dots represent the single-epoch ISB series and the red line denotes the mean value of one-week ISB solution, which is the same for all the figures below. It can be seen that both the code and phase ISBs are very stable despite of the random terms. The Standard Deviation (STD) of code ISBs is at decimetre level and much larger than that of phase ISBs (within 0.01 cycles), which is mainly caused by the pseudorange noises. In addition, the mean value of all the ISBs is extremely close to zero, except for the L1-E1 code ISB (about 10 cm, but smaller than pseudorange noises). Thus GPS-Galileo ISB parameters can be ignored in tightly-coupled DD model for baselines consisting of identical receiver types.

Figures 3 and 4 present the time series of GPS-Galileo (L1-E1 and L5-E5a) code and phase ISBs, respectively, for the baseline with different receiver types: ‘SEPTENTRIO-TRIMBLE’ and ‘JAVAD-TRIMBLE’. For the zero-baseline ‘CUT0-CUT1’, periodic variations appear in the code ISB and the amplitudes of the wave are below 1 m. STD of phase ISBs (about 0.005 cycles) is consistent with that of the baseline with the same receiver pair. Moreover, STD of both the code and phase ISBs for the ultra-short baseline ‘CUT0-CUAA’ is much larger than that for the zero-baseline ‘CUT0-CUT1’. The significant variations are due to the multipath effect which can only be fully eliminated in zero-baseline. In general, both the code and phase ISBs appear stable with time and can be calibrated by a priori correction.



**Figure 3.** Time series of GPS-Galileo (L1-E1 and L5-E5a) code ISBs for the baselines with different receiver types: ‘SEPTENTRIO-TRIMBLE’ and ‘JAVAD-TRIMBLE’

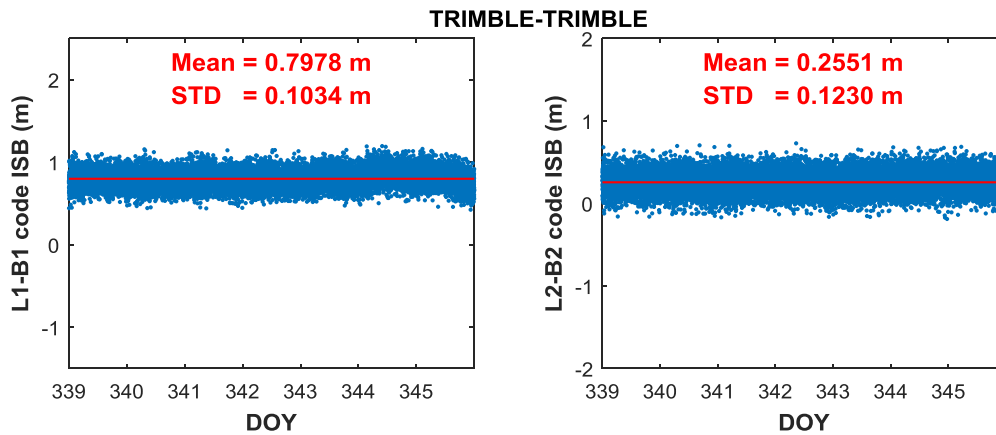




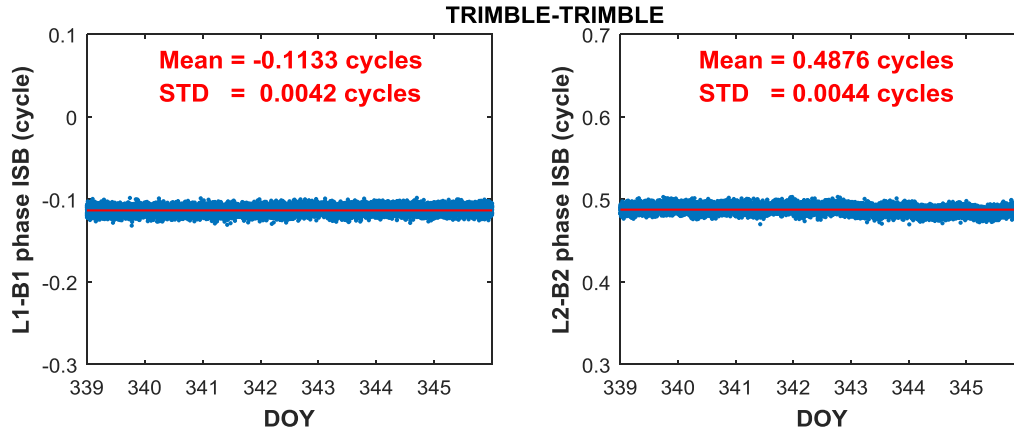
**Figure 4.** Time series of GPS-Galileo (L1-E1 and L5-E5a) phase ISBs for the baselines with different receiver types: 'SEPTENTRIO-TRIMBLE' and 'JAVAD-TRIMBLE'

#### 4.3 GPS-BDS DD ISB Stability Analysis

Figures 5 and 6 demonstrate the time series of GPS-BDS (L1-B1 and L2-B2) code and phase ISBs, respectively, for the baseline with the same receiver type: 'TRIMBLE'. It can be found that both the code and phase ISBs still have long-term stability, but their mean value is non-zero. Thus GPS-BDS ISB parameters also need to be taken into account in tightly-coupled DD model for baselines consisting of identical receiver types. As more BDS satellites can be employed in ISB estimation, the STD of GPS-BDS ISBs is smaller than that of GPS-Galileo ISBs and the time series appear more stable.

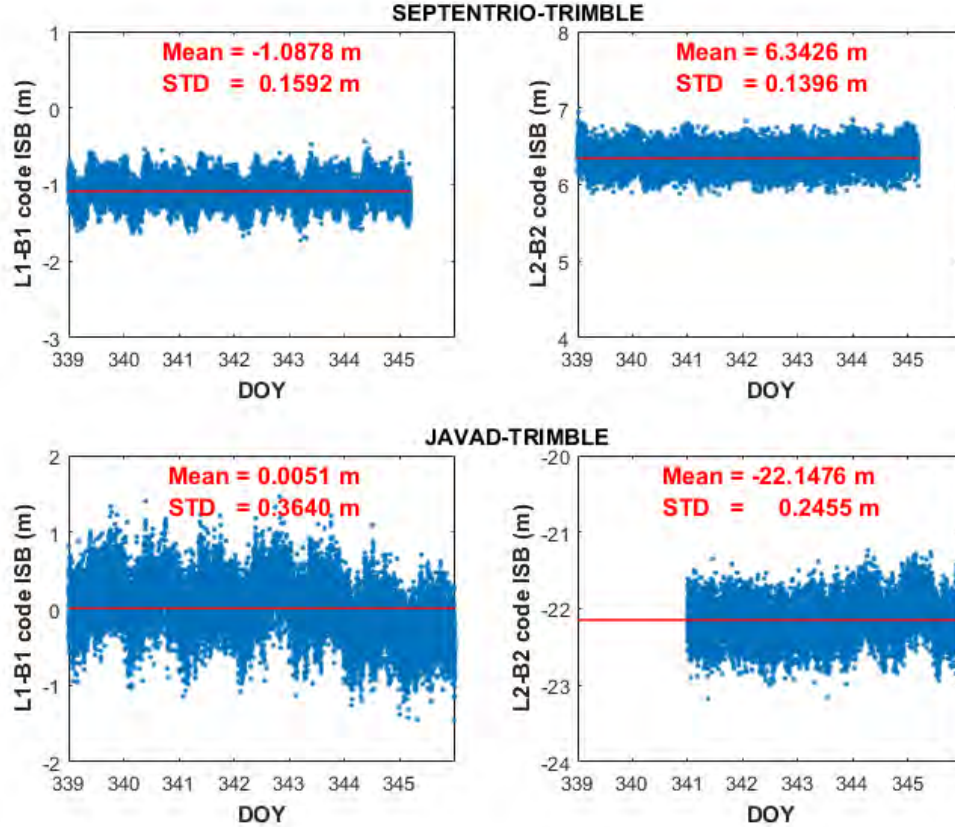


**Figure 5.** Time series of GPS-BDS (L1-B1 and L2-B2) code ISBs for the baseline with the same receiver type: 'TRIMBLE'

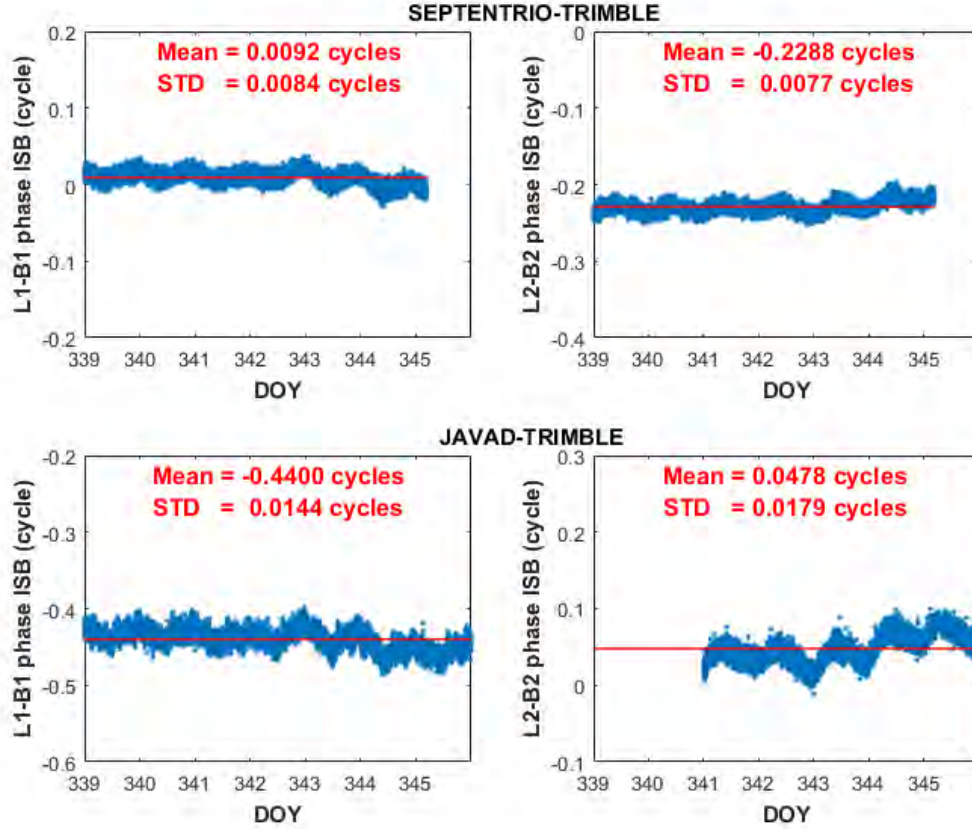


**Figure 6.** Time series of GPS-BDS (L1-B1 and L2-B2) phase ISBs for the baseline with the same receiver type: ‘TRIMBLE’

Figures 7 and 8 illustrate the time series of GPS-BDS (L1-B1 and L2-B2) code and phase ISBs, respectively, for the baseline with different receiver types: ‘SEPTENTRIO-TRIMBLE’ and ‘JAVAD-TRIMBLE’. The significant ISB variations in the baseline ‘CUT0-CUAA’ are also due to the multipath effect. It is obvious that both the code and phase ISBs have small periodic variations besides the random terms. It can be seen that in the zero-baseline ‘CUT0-CUT1’, the amplitudes of the wave are within 0.5 m and 0.03 cycles for code and phase ISB accordingly, by contrast, in the ultra-short baseline ‘CUT0-CUAA’, the amplitudes of the wave are within 1 m and 0.05 cycles for code and phase ISB, respectively. In the real-time estimation of GPS-BDS ISB, a random walk process can be utilized to model this variation.



**Figure 7.** Time series of GPS-BDS (L1-B1 and L2-B2) code ISBs for the baselines with different receiver types: ‘SEPTENTRIO-TRIMBLE’ and ‘JAVAD-TRIMBLE’



**Figure 8.** Time series of GPS-BDS (L1-B1 and L2-B2) phase ISBs for the baselines with different receiver types: ‘SEPTENTRIO-TRIMBLE’ and ‘JAVAD-TRIMBLE’

## 5. CONCLUSIONS

In this paper, we have proposed the procedure to estimate multi-GNSS code and carrier phase ISBs towards the goal to resolve the DD integer ambiguities between any satellites within the multi-constellation GNSS. We investigate the magnitude and stability of DD ISBs for both overlapping and mixed frequencies of different GNSS constellations using identical and different receiver types. For a baseline with receivers of the same type, it is verified that ISBs have long-term stability for both overlapping and mixed frequencies. The GPS-Galileo ISBs can be neglected, but the GPS-BDS ISBs need to be calibrated in tightly-coupled DD model. For a baseline with different receiver types, the GPS-BDS ISBs may have small variations which can be properly modelled in real-time estimation.

The next step of our research will focus on multi-GNSS ISB estimation in undifferenced and network mode, in order to realize reliable tightly-coupled multi-GNSS PPP-RTK.

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