A New Model for Precise Point Positioning to Improve Fault Detection

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Contents

• Problem statement & proposed solution.
• A new observation model in RT-PPP
• Characterisation of RT orbital and clock corrections
• FDE with the new model
• Prediction of corrections to replace excluded ones
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Problem statement?

- RT-PPP is used in natural hazard warning systems and intelligent transport systems.
- Current RT-PPP models combine the observations with the orbit and clock corrections in one term.
- Faults in these corrections (e.g. due to spoofing or meaconing) will result in exclusion of satellite measurements, which would degrade positioning.
- It may disable RT-PPP.
Application of IGS-RTS orbit corrections

\[
\delta \tilde{\rho} = [\delta \rho_r \ \delta \rho_a \ \delta \rho_c]^T + [\delta \dot{\rho}_r \ \delta \dot{\rho}_a \ \delta \dot{\rho}_c]^T (t - t_o)
\]

\[
x^s = x^{brdcst} + \text{diag}(e_r \ e_a \ e_c) \ \delta \tilde{\rho}
\]

\[
c \ \delta t = q0 + q1 \ (t - t_o) + q2 \ (t - t_o)^2
\]

\[
c \ \delta t^s = c \ t^{brdcst} + c \delta t
\]
Solution

• Treats the corrections to broadcast orbit and clock corrections as quasi-observations.

• This model enables fault detection and exclusion of these corrections separate from the observations.

• The excluded faulty corrections can be replaced by predicted values.

• The method preserves positioning by keeping measurements with faulty corrections, and use them with predicted corrections.

• The second advantage: it includes both the variance and covariance information of the precise orbits and clock corrections.
New observation equations

\[ P^s_r = \rho^s_r + E C^s_r + c \overline{d t}_r + \mu^s_r T + \varepsilon_{P^s_r} \]

\[ \phi^s_r = \rho^s_r + E C^s_r + c \overline{d t}_r + \mu^s_r T + \lambda \bar{N}^s_r + \varepsilon_{\phi^s_r} \]

\[ \bar{E} C^s_r = E C^s_r + \varepsilon_{\bar{E} C^s_r} \]

\[ E C^s_r = \cos(\theta) d \rho^s - c \overline{d t}^s \quad \cos(\theta) = \frac{(x^s - x^{\text{bdct}}). (x^s - x_u)}{||x^s - x^{\text{bdct}}|| \cdot ||x^s - x_u||} \]

\[ y = H x + e \]

\[ \begin{pmatrix} P^s_r \\ \phi^s_r \\ \bar{E} C^s_r \\ y \end{pmatrix} = \begin{pmatrix} G & u & \mu & 0 & u \\ G & u & \mu & \lambda I & u \\ 0 & 0 & 0 & 0 & u \\ H \end{pmatrix} \begin{pmatrix} X_u \\ c \overline{d t}_r \\ T \\ \bar{N}^s_r \\ E C^s_r \end{pmatrix} + \varepsilon \quad \text{for } s = 1 \text{ to } n \]
Characterisation of the orbital and clock corrections

- Observation errors in the open environment are assumed normally distributed.

- **Can orbital and clock corrections assumed normally distributed?**

- Six months of IGS-RTS GPS orbital and clock corrections (1-6, 2016) of the IGC01 stream were analysed

- Errors in the IGS-RTS corrections computed by referencing the resulting orbits and clocks to IGS final products.

- Normality tests + Histograms, Q-Q and CDF plots
Characterisation of clock corrections

Histograms

Q-Q

CDF

PRN16

PRN 29

PRN30
Characterisation of orbit corrections

Histograms

Q-Q

CDF

PRN16  PRN 29  PRN30
Accuracy of IGS-RTS products

Study period (January-June 2016).

<table>
<thead>
<tr>
<th>Orbital 3D correction (m)</th>
<th>Clock corrections (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean error</strong></td>
<td><strong>Mean error</strong></td>
</tr>
<tr>
<td>(of absolute values)</td>
<td>(of absolute values)</td>
</tr>
<tr>
<td><strong>Mean error</strong></td>
<td><strong>RMSE</strong></td>
</tr>
<tr>
<td>0</td>
<td>0.041</td>
</tr>
<tr>
<td>0.023</td>
<td>0</td>
</tr>
<tr>
<td>0.281</td>
<td>0.210</td>
</tr>
</tbody>
</table>
Flowchart of FDE testing and data fusion

GNSS observations

Received orbit and clock corrections

Predicted orbit and clock corrections

Quasi-obs.

Fault detection test

Fault detection test

Identification test

Identification confirmation

Pass

Fail

Remaining obs. after identification

Identified corr.

Quasi-obs. (replacement of faulty corrections)

Process the data

Received orbit and clock corrections

\[ \hat{\mathbf{t}}^T Q_{\hat{\mathbf{t}}}^{-1} \hat{\mathbf{t}} \geq \chi^2_{a_1}(df, 0) \]

\[ |w_j| \geq N_{a_2}(0,1) \frac{\sigma_{\hat{z}_{ij}}}{2} \geq N_{a_2}(0,1) \]

\[ |\hat{x} - \hat{x}_{ij}| \frac{\sigma_{\hat{z}_{ij}}}{2} \geq N_{a_2}(0,1) \]

IGNSS 2018, 7-9 Feb 2018, Sydney
Prediction model of the replacement clock corrections

\[ \delta t = a_0 + a_1 \Delta t + \frac{a_2 \Delta t^2}{2} + \sum_{i=0}^{k} A_i \sin \left( \frac{\Delta t_i}{\lambda_i} \times 2\pi + \frac{t_{\phi_i}}{\lambda_i} \times 2\pi \right) + \varepsilon_{\delta t} \]

- Polynomial + k sinusoidal periods.
- Polynomial: \( a_0, a_1 \) and \( a_2 \) (bias, drift & drift rate) – cyclic \( A_i \) \& \( t_{\phi_i} \) determined using least squares,
- \( \lambda_i \) from FFT

weight is assumed decaying gradually with time

\[ W = diag(w_j), \quad w_j = e^{-\Delta t/T} \times 1/\sigma_{\delta t}^2 \]

- user sequentially build the prediction model with a sliding time window and use predicted whenever real-time clock corrections are unattainable.
Accuracy of prediction of the clock corrections

**GPS**: errors < ±0.25 ns for after 0.5 hr, error < ±0.5 ns after 1 hr

**GLONASS**: errors < ±1 ns for after 0.5 hr, error < ±2 ns after 1 hr

**QZSS**: error < ±0.05 ns after 1 hr, error < ±0.1 ns after 1 hr
Validation

- Proposed method was validated at five IGS stations.
- Apply PPP with float ambiguities.
- Results were compared to results of the traditional PPP
- **Assumed a spoofing attack.** Artificial faults were inserted at random events.
- The test was repeated with a varying number of faults

![Artificial errors vs Time](image)
Positioning Accuracy

2/1/2017

Station PERT (Australia)

3/1/2017

Station PTBB (Germany)

4/1/2017
# Average positioning errors over 3 days

<table>
<thead>
<tr>
<th>Number of faulted corrections</th>
<th>Station PERT</th>
<th>Station PTBB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Proposed method</td>
<td>Traditional PPP</td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>N</td>
</tr>
<tr>
<td>No faults</td>
<td>0.033</td>
<td>0.034</td>
</tr>
<tr>
<td>1 satellite</td>
<td>0.035</td>
<td>0.034</td>
</tr>
<tr>
<td>2 satellites</td>
<td>0.044</td>
<td>0.070</td>
</tr>
<tr>
<td>3 satellites</td>
<td>0.061</td>
<td>0.064</td>
</tr>
<tr>
<td>All sats</td>
<td>0.102</td>
<td>0.103</td>
</tr>
</tbody>
</table>
Positioning Errors

2/1/2017

Positioning Accuracy (m)

No. of sats

Time (hr)

2/7/2017

Positioning Accuracy (m)

No. of sats

Time (hr)

HERT (UK)  
GMSD (Japan)  
SASK (Canada)
### Average positioning accuracy for all sites

<table>
<thead>
<tr>
<th></th>
<th>Proposed method</th>
<th></th>
<th>Traditional PPP</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E</td>
<td>N</td>
<td>U</td>
<td>E</td>
</tr>
<tr>
<td>No faults</td>
<td>0.085</td>
<td>0.094</td>
<td>0.110</td>
<td>0.086</td>
</tr>
<tr>
<td>1 satellite</td>
<td>0.097</td>
<td>0.104</td>
<td>0.135</td>
<td>0.102</td>
</tr>
<tr>
<td>2 satellites</td>
<td>0.145</td>
<td>0.155</td>
<td>0.193</td>
<td>0.201</td>
</tr>
<tr>
<td>All sats</td>
<td>0.210</td>
<td>0.199</td>
<td>0.212</td>
<td>N/A</td>
</tr>
</tbody>
</table>
Conclusion

- A new model is presented to detect and solve the effect of faults (e.g. by spoofing) in orbital and clock corrections.

- The detected faulty corrections are replaced by predicted values.

- The prediction model is built every 10-15 minutes, with a sliding time window.

- Both the proposed method and traditional PPP were able to detect the faults, the conventional PPP positioning accuracy degraded sharply as the number of outliers increases since more observations were excluded.

- With the proposed method, positioning was maintained all the time with accuracy within 0.20m.
Thank you

Questions